# QUANTUM COMPUTATIONS AND IMAGES RECOGNITION

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**Abstract**

The using of quantum parallelism [1, 2] is often connected with consideration of quantum system with huge dimension of space of states. The $n$–qubit register can be described by complex vector with $2^n$ components (it belongs to $n$'th tensor power of qubit spaces). For example, for algorithm of factorization of numbers [3] by quantum computer $n$ can be about a few hundreds for some realistic applications for cryptography. The applications described further are used some other properties of quantum systems and they do not demand such huge number of states.

The term *"images recognition"* is used here for some broad class of problems. For example, we have a set of some objects $V_i$ and function of "*likelihood*":

$$F(V,W) < F(V,V) = 1; \quad (V \neq W).$$

If we have some "noisy" or "distorted" image W, we can say that recognition of $W$ is $V_i$ if $F(W,V_i)$ is near 1 for some $V_i$.

On the other hand, the description of measurement process in quantum mechanics describes probability of registration as $\Pr = |(V,W)|^2$, there vector $V$ describes the system, $W$ - measurement device, and $(V,W)$ is a scalar product. So, some kind of "likelihood" function is ready to use in quantum computers. Of course, the simplest examples are Stern-Gerlach experiment for electrons and polarizer for photons.

In previous work [4] as an example of using quantum computers for some applied problem was chosen simple expert system. There are many other similar approaches was found in different areas later. It is associative memory [5], fuzzy logic [6], artificial neural networks,*etc.*. These models are used for further research of the subject.

# 1 Introduction

Richard Feynman [7] has compared using of continuous properties of quantum systems for building of quantum computers with using of transistors only as two-state systems in electronic digital computer. On the other hand, *analogue computers* use continuous properties of physical systems for modeling. In the paper is considered some kind of "*analogue*" quantum computers those use properties of quantum systems for building linear models for data analysis[1]. The digital computers are much more common and convenient due to *universality*, but quantum systems also possess some kind of universality [1]. There is also some possibility that the same quantum computer could implement both analogue modelling discussed further and parallel digital computation.

There are exist few examples of using some models of classical physics for advanced processing of information — algorithms based on models of statistical physics are widely used for data analysis, image recognition [9], learning of artificial neural networks, *etc.*.

The some examples of using *quantum* systems for particular tasks like *image analysis* are discussed further. Linear operators and vector spaces are not only usual language of quantum mechanics, but they are used also in many works devoted to such AI problems as associative memory, image recognition, parallel distributive processing, *etc.*. Due to such properties the quantum systems can be used for *analogue* computing of different linear models in data analysis.

The main problem of using of quantum systems is to choose the algorithms of modeling those are compatible with quantum laws. For example many works devoted to artificial neural network use noncompact space like $\mathbf{R}^n$ and some nonlinear functions together with linear operators. Such models are seem hardly compatible with language of quantum mechanics. The impossibility of getting the full information about a quantum system by measurement and the other specific properties of quantum systems are also must be taken into account. On the other hand, the more appropriate models we are using for *analogue* modeling by some quantum system, the more elementary and simple system we can use. It is useful for design of very *miniature* and *fast* quantum devices.

# 2 The spaces of images

The space of images is usually described as some vector space $\mathbf{V}$. For example, a monochrome digital picture $n \times m$ can be described as a set of $N = n\,m$ real numbers corresponding to the intensity of light in each *pixel* (picture element), *i.e.* some vector $\mathbf{x} \in \mathbf{R}^N$ [5, 9].

The *hyperspheres* $\{\mathbf{S}^k : \mathbf{x} \in \mathbf{R}^{k+1}, \|\mathbf{x}\| = 1\}$ and *projective spaces* (spaces of rays) $\{\mathbf{R}P^k : [\mathbf{x}] \in \mathbf{R}^{k+1}_{\setminus\{0\}}/\mathbf{R}_{\setminus\{0\}};\ [x_0{:}x_1{:}\ldots{:}x_k] = [\lambda x_0{:}\lambda x_1{:}\ldots{:}\lambda x_k]; \mathbf{x} \neq \mathbf{0}, \lambda \neq 0\}$ are also can be used.

For example, in case of monochrome picture we can multiply all intensities on the same nonzero positive value and the picture does not change. Due to such invariance we can use only vectors with unit length and space of images is subspace of the sphere $\mathbf{S}^{N-1}$.

[1] Of course, the *quantum parallelism* also uses whole state space of quantum system

An *image recognition* can be described as following task. There are $m$ *known* images: $\mathbf{v}^{(1)} \ldots \mathbf{v}^{(m)} \in \mathbf{V}$. A system should "*recognize*" any image $\mathbf{v}^{(i)}$ by his *noisy* or *incomplete* version $\mathbf{w}$. If the $\mathbf{w}$ is not precisely equal to some $\mathbf{v}^{(i)}$ we can use a measure on $\mathbf{V}$ to choose $\mathbf{v}^{(i)}$ that more "close" to $\mathbf{w}$. Often such a measure is *Euclidean distance* on $\mathbf{R}^n$:

$$\|\vec{w} - \vec{v}\|_{R^n} \equiv \sqrt{\sum_{i=1}^{n} (w_i - v_i)^2} \tag{1}$$

In case of $\mathbf{S}^{n-1}$ the *cosine* of angle between two vectors with unit length is:

$$\cos(\angle_{\vec{w},\vec{v}}) = \sum_{i=1}^{n} w_i v_i \equiv (\vec{w}, \vec{v}) \tag{2}$$

The $(\mathbf{w}, \mathbf{v})$ is scalar product and for vectors with unit norm:

$$(\vec{w}, \vec{v}) = 1 \Longleftrightarrow \vec{w} = \vec{v} \tag{3}$$

The property (3) is used in some approaches for image recognition. For space of images like $\mathbf{S}^k$ criterion $1 - \varepsilon < (\mathbf{w}, \mathbf{v}^{(i)}) \leq 1$ can be used. It should be mentioned that the measures are good for models of errors like addition of some random noise or lack of some areas in picture. The same image after *moving* or *rotation* may have very low correlation with initial image from point of view of above mentioned criterion.

It should be mentioned that the more *homogeneously* distribution of images in the space the better. For example we can subtract *half of average intensity* from any points of the monochrome picture before normalization

$$y_i = x_i - \frac{1}{2N} \sum_{i=1}^{N} x_i, \quad \vec{z} = \frac{\vec{y}}{\|\vec{y}\|} \tag{4}$$

In case of homogeneous distribution on the $\mathbf{S}^N$ the scalar product of two random vectors is:

$$(\vec{w}, \vec{v}) \sim N^{-1/2} \stackrel{N\to\infty}{\longrightarrow} 0 \tag{5}$$

If we are going to use quantum systems for analogue modeling, then using of approaches with $\mathbf{S}^k$ or $\mathbf{R}P^k$ is especially justified.

# 3 The Hilbert spaces

The space of states of quantum system is described by *Hilbert space* $\mathcal{H}$, *i.e.* complex vector space with Hermitian scalar product:

$$(\mathbf{a}, \mathbf{b}) = \sum_{i=1} a_i \overline{b_i}\ , \quad \|\mathbf{a}\| = \sqrt{(\mathbf{a}, \mathbf{a})} \tag{6}$$

In physics there are notations $|a\rangle$ for the vector $\mathbf{a}$ and $\langle a|$ for co-vector $\mathbf{a}^*$, and

$$\langle b \mid a \rangle = \mathbf{b}^* \mathbf{a} = (\mathbf{a}, \mathbf{b}). \tag{7}$$

The vectors $|\psi\rangle$ and $\lambda|\psi\rangle$ for any $\lambda \in \mathbf{C} - \{0\}$ are describe the same physical state. The states of quantum systems are rays in complex vector space *i.e.* points in *complex projective space* $\mathbf{C}P^n$ (or $\mathbf{C}P^\infty$). Due to projectivity, we can consider only states with unit norm $\|\psi\| = 1$. It is hypersphere $\mathbf{S}^{2n+1} \subset \mathbf{R}^{2n} \simeq \mathbf{C}^n$. The vectors have property maximum projection (eq. 3) for scalar product (eq. 6), but for the same physical state $|\psi\rangle$ and $|\psi'\rangle = e^{i\varphi}|\psi\rangle$: $\langle\psi \mid \psi'\rangle = e^{i\varphi} \neq 1$.

The different quantum states correspond to space of rays $\mathbf{C}P^n \simeq \mathbf{S}^{2n+1}/\mathbf{S}^1$. The analogue of property (3) for space of states of quantum system is:

$$\Pr(\chi \to \psi) \equiv |\langle\psi \mid \chi\rangle|^2 = 1 \Longleftrightarrow |\chi\rangle = \lambda|\psi\rangle \Longleftrightarrow [\psi] = [\chi] \tag{8}$$

Where $[\psi], [\chi] \in P(\mathcal{H})$ are complex *rays* in Hilbert space those correspond to states of quantum system.

# 4 Quantum systems

Let us consider physical meaning of the algebraic formulae above. The $\Pr(\chi \to \psi)$ is probability of registration of system $|\chi\rangle$ by the measurement device that works as a filter for state $|\psi\rangle$ [8]. It is some kind of binary Yes / No consideration. We are "asking", does the system in state $|\psi\rangle$, and for quantum system $|\chi\rangle$ we have "answer" Yes with probability $\Pr(\chi \to \psi) = |\langle\psi \mid \chi\rangle|^2$. We can see, that $\Pr = 1$ *if and only if* $|\chi\rangle$ and $|\psi\rangle$ is the same physical state *i.e.* the same point on complex projective space. There is second model of measurement that also will be useful further. If we have some orthogonal basic states $|\psi_i\rangle$ and quantum system $|\chi\rangle$

$$\|\chi\| = 1, \quad \langle\psi_i \mid \psi_j\rangle = \delta_{ij}, \quad |\chi\rangle = \sum_{i=1}^{n} a_i|\psi_i\rangle,\ a_i = \langle\chi \mid \psi_i\rangle. \tag{9}$$

then we can get one of $|\psi_i\rangle$ due to measurement of the $|\chi\rangle$ with probability $\Pr_i = (a_i)^2$, *i.e.* have got one of $n$ versions of outcome instead of two in the previous example. But $|\psi_i\rangle$ in this case must be *orthogonal*. This simple quantum mechanical idea has emphasized here for further discussion about using of *orthogonal* set of images.

The consideration of particular problems of data analysis that can be related with the mentioned approach are followed. Further will be used examples of data analysis with compact space of images like $\mathbf{R}P^n$, $\mathbf{S}^n$ that was developed by different authors. Such kind of models can be considered as a good starting point due to analogy between conditions of maximum of (3) for real spaces and (8) for complex space of state. It should be mentioned also that $\mathbf{R}P^n \subset \mathbf{C}P^n$ and so an $v \in \mathbf{R}P^n$ can be treated as some formally *possible* state of quantum system *i.e.* it is not contradict with laws of quantum mechanics.

# 5 The quantum systems and analogue image recognition

The using of classical device for analogue calculation of the expressions like (eq.2) for scalar products [5] was more justified before the *digital computers* have become fast and cheap enough. The quantum computers could make the such approach useful again.

For realization of the above mentioned algorithms of image recognition we should add to usual unitary operations of quantum computer a new one. The operation is similar to *measurement* in usual description of macroscopic experiment [8]. It can be described as *transition* from state $|\psi\rangle$ to state $|\chi\rangle$ with probability $|\langle\chi \mid \psi\rangle|^2$. In description of quantum computers the related effects are usually considered as undesirable sources of errors[2]. Here is discussed an useful application of the effects.

Let us consider some method of *input* of the images data to quantum computer as smooth map from the space of images to the space of states of quantum system: $\mathcal{I} : \mathbf{V} \to \mathcal{V} \subset \mathcal{H}$. [3]

In this case simple quantum *read-only memory* (q-ROM) for *one* image can be considered as a filter $|\psi_{image}\rangle\langle\psi_{image}|$ that receive some $|\psi\rangle$ as an input and produce $|\psi_{image}\rangle$ as the output with probability $\Pr = |\langle\psi_{image} \mid \psi\rangle|^2$ that is equal to *one* if $|\psi\rangle = |\psi_{image}\rangle$.

If distribution of the inputs is *homogeneous* and dimension of state space $N$ is big enough, then scalar product is "close" to *zero* for some arbitrary input due to (eq. 5). Because of this property the probability of recognition is very small for arbitrary images ($\Pr \sim 1/N$). On the other hand, for images that differ from true image due to some small errors the recognition is near to *one*.

If we have a *classical input* like monochrome picture then there is possibility of repeatedly preparing of the same $|\psi(input)\rangle$ to satisfy desirable statistical criteria for any number of different q-ROMs with different images that would be enough to find an image with maximum probability $\Pr_i = |\langle\psi(input) \mid \psi(image_i)\rangle|^2$. On (Fig. 1) is shown source of quantum systems controlled by some *input*. The $|\chi_{input}\rangle$ can be considered for example as molecular beam that is split by some partially transparent passive "mirrors" to $n$ arms with filters $\iota_1 \ldots \iota_n$. If the intensities before the filters in each arm are the same ($I_0/n$) then intensities of the beams after filters are $I_k = |\langle\iota_k \mid \chi_{input}\rangle|^2 I_0/n$. The channel that correspond to restored image has maximum intensity.

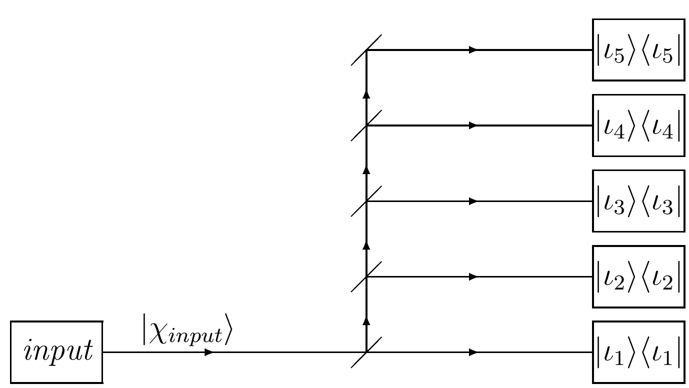


Figure 1: Splitting of dense beam

The more effective procedure can be used in case of *orthogonal* images. If the images

[2]The same expression $|\langle\chi \mid \psi\rangle|^2$ is called *fidelity* of quantum channel [10, 11]

[3]Here *smooth* means *continuous* from respect of some norms on $\mathbf{V}$ and $\mathcal{H}$.

is not quite orthogonal, but they have scalar products near zero (see eq. 5) it is possible to use standard method of ortogonalization of vectors and use new orthogonal set of corrected images. The quality of such methods for real problems of images recognition is discussed in [5].

Let us suppose that we have such a set of orthogonal images. Then it is possible to make measurement that recognize any of $k$ images with $\Pr = 1$. The such measurement is described by (eq. 9) if all images correspond to some basic vectors $|\psi_i\rangle$ in (eq. 9). For example it may be first $k$ vectors $|\psi_1\rangle \ldots |\psi_k\rangle$, $k \ll n$.

The simple example is shown on (Fig. 2). Here the $U_r$ is unitary operator that rotate an orthogonal set of images to orthogonal set of basic vectors of measurement device. By using different $U_r$ it is possible to make q-ROM for any $n$ orthogonal vectors and first $k \ll n$ can be good approximation for set of images due to conditions like (eq. 5).

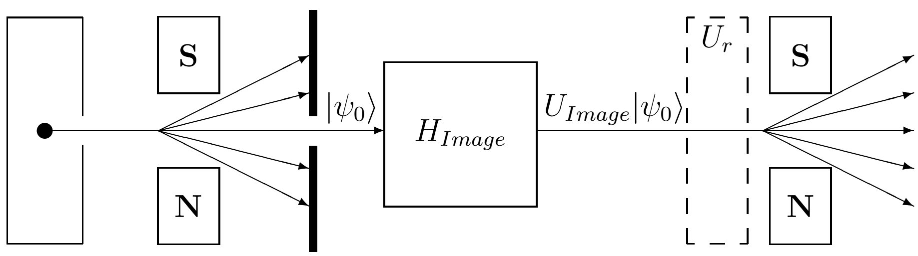


Figure 2: Recognizing of orthogonal images

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